2010 AB
1

$f(t) = 7te^{\cos t}$ cu ft/hr [midnight, 9am]

$g(t) = \begin{cases} 0 & [0,6) \\ 125 & [6,7) \\ 108 & [7,9] \end{cases}$ removal

a) how many ft have accumulated by 6 am

Ans: total accumulation = starting amount + total add − total removed

$0 + \int_0^6 f(t)\,dt - \int_0^6 g(t)\,dt$

$= \int_0^6 7te^{\cos t}\,dt - \int_0^6 0\,dt = 142.27\overset{5}{4}$ +1 +1

b) Find the rate of change of volume of snow on driveway @ 8 am.

$= f(8) - g(8) = 56e^{\cos 8} - 108$ +1

c) $h(t)$ = total amt of snow removed.

$h = \begin{cases} 0 & 0 \le t < 6 \quad +1 \\ 125(t-6) & 6 \le t < 7 \quad +1 \\ 125 + 108(t-7) & 7 \le t \le 9 \quad +1 \end{cases}$

d) cubic ft on driveway @ 9 am?

Ans: total accumulation = starting amount + total add − total removed

$0 + \int_0^9 f(t)\,dt - \int_0^9 g(t)\,dt$

$= \int_0^9 7te^{\cos t}\,dt - 125 - 216$ +1 +1

$= 26.33\underset{5}{4}$ cu ft +1

Apr 22-8:17 AM

2)

t (hrs)	0	2	5	7	8
$E(t)$ (hundreds of entries)	0	4	13	21	23

noon: $t=0$; $E(t)$ is a differentiable f^n^

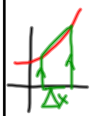
a) approximate the rate [[100s entries / hr]] at which entries were being deposited at $t=6$.

Approximate instantaneous r.o.c. with average r.o.c

rate of chg @ 6pm $\approx \dfrac{E(7)-E(5)}{7-5} = \dfrac{21-13}{2} = 4$ (+1) [hundreds of ent per hr]

b) approx. the value of $\frac{1}{8}\int_0^8 E(t)\,dt$ with trap sum. & expl

area of trapezoid: $\frac{1}{2}\left(f(x_n)+f(x_{n+1})\right)\Delta x$



$\frac{1}{8}\int_0^8 E(t)\,dt \approx \frac{1}{8}\Big[\frac{1}{2}(4+0)(2) + \frac{1}{2}(13+4)(3) + \frac{1}{2}(21+13)(2) + \frac{1}{2}(23+21)(1)\Big] = 10.6$ or 10.6 (+1)

what does $\frac{1}{8}\int_0^8 E(t)\,dt$ represent? (+1)

+1 $\int_0^8 E(t)\,dt$ is the "total" or sum of all the values of $E(t)$

$\frac{1}{8}\int_0^8 E(t)\,dt$ is the average value of $E(t)$.

In other words, the average number of entries (in hundreds of entries) that were in the box from t 0 → 8

c) process ["remove"] entries by $P(t)=t^3-30t^2+298t-9$ (hundreds of ent) [[$t\in[8,12]$]]. how many entries are left @ 12

total entries left = starting + total-in − total-out

$= 0 + E(8) - \int_8^{12} t^3-30t^2+298t-9$ (+1)

$23-16 = 7$ (+1) (hundred entries left)

d) entries "processed most quickly"?

i.e. the rate of processing entries is a maximum

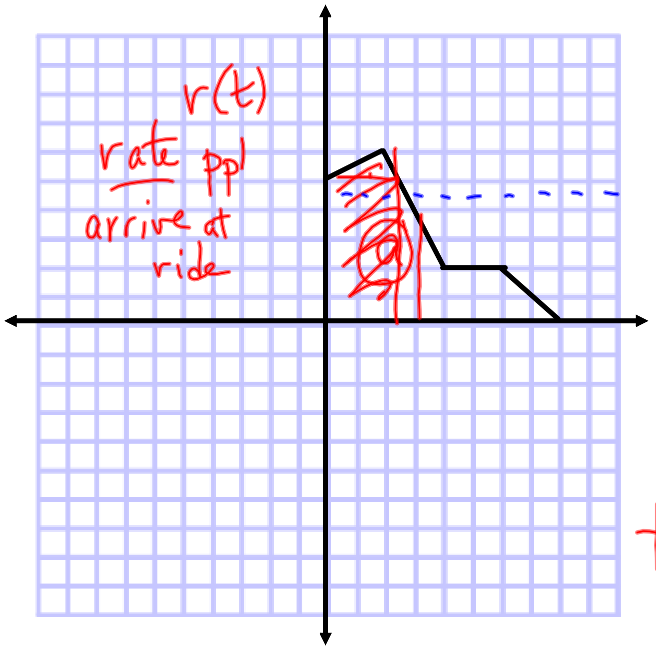
$P(t)=t^3-30t^2+298t-976$

to find a maximum

$P'(t)=3t^2-60t+298=0$

$t=\dfrac{60\pm\sqrt{(-60)^2-4(3)(298)}}{6} =$ $t=9.183$, $t=10.816$

Apr 22-8:38 AM



ppl on line at $t=0$: 700

rate at which ppl move onto ride : 800 ppl / hr

a) how many ppl arrive between 0 and 3?

$$\text{total arrivals} = \int_0^3 \text{instantaneous arrivals}$$

$$= \int_0^3 r(t)\,dt \;\; +1 \;\; = \tfrac{1}{2}(1000+1200)(2) + \tfrac{1}{2}(1200+800)(1) \;\; +1$$

$$= (3200 \text{ ppl})$$

unnecessary

b) # of ppl waiting in line increasing or decreasing between 2 and 3?

INCREASING. the rate is positive. +1

c) when is the wait the longest?

i) at $t=3$. +1

ii) $700 + 3200 - 3(800) = 1500$ ppl +1

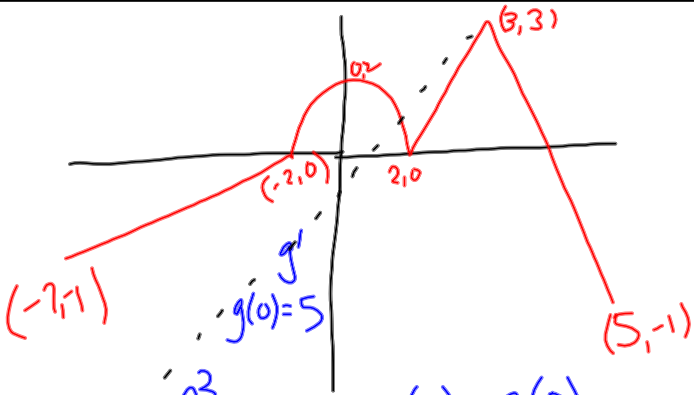
iii) after 3, ppl arrive more slowly than they enter the ride.

$r(t) < 800$ when $t > 3$ +1

d) the earliest time at which the[re] is no longer a line.

$$0 = 700 + \int_0^t r(x) - 800t$$

5)



g(3): $\int_0^3 g'(x)\,dx = g(3) - g(0)$

$\int_0^3 g'(x)\,dx = g(3) - 5$

Area quarter circle + Area triangle f 2-3 $= \frac{1}{4}\pi(2)^2 + \frac{1}{2}(1)(3) = g(3) - 5$

$\pi + \frac{3}{2} + 5 \quad = g(3)$

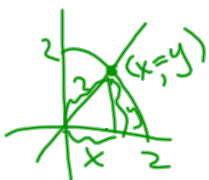
g(-2) . . . -

b) a point of inflection occurs
- when concavity changes
- i.e. when f'' changes sign — when f''=0 / when f'' undefined
- i.e. when increasing/decreasing property of f' changes
- i.e. at x=0, 2, 3

c) $h(x) = g(x) - \frac{1}{2}x^2$

find critical pts [i.e. when f'=0 or f'' und]

$h'(x) = g'(x) - x$



$g'(x) - x = 0$

$g'(x) = x$ i.e. when $x = +\sqrt{2}$, $x = 3$

$x^2 + y^2 = 2^2$

$2x^2 = 2^2 = 4$

$x^2 = 2 \quad x = +\sqrt{2}$

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